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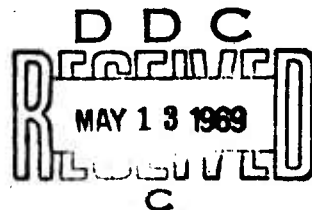
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THE EXTENT OF LAMINAR, TWO-DIMENSIONAL BOUNDARY LAYER SEPARATION DUE TO A COMPRESSION CORNER AT MODERATELY HYPERSONIC SPEEDS

KARLHEINZ O. W. BALL

SYSTEMS RESEARCH LABORATORIES, INC.
DAYTON, OHIO

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FOREWORD

This technical report was initiated while the author was on active duty with the United States Air Force, stationed at the Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio. The initial work was prepared at the Hypersonic Research Laboratory under Project No. 7064, entitled "High Velocity Fluid Mechanics". The report was completed at Systems Research Laboratories, Inc., Dayton, Ohio, and was sponsored by the Fluid Dynamics Facilities Research Laboratory of the Aerospace Research Laboratories under Contract No. AF33(615)-2175, Project No. 7065, "Aerospace Simulation Techniques Research".

ABSTRACT

A simple technique is presented for estimating the extent of separation of a two-dimensional laminar boundary layer including effects of Mach number, Reynolds number, and heat transfer. Quantitative effects of these parameters are discussed for moderately hypersonic speeds.

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LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

H	form factor $\delta^*_{tr}/\theta_{tr}$
K	pressure gradient parameter
K_0, K_2	temperature parameters, see Eq. (6)
L	length parameter
M	Mach number
Re	Reynolds number
T	temperature
x	streamwise surface coordinate
X	extent of separation
β	L ratio between separated and attached flows
δ^*	boundary layer displacement thickness
θ	boundary layer momentum thickness
Δ	difference, i.e., initial minus final condition
Θ_F	compression surface deflection angle
λ	constant of proportionality
$\bar{\chi}$	viscous interaction parameter

Subscripts

1	beginning of separation interaction
HL	hinge line
i	incipient condition
tr	transformed
w	wall
o	stagnation

SECTION I

INTRODUCTION

Laminar, two-dimensional boundary layer separation has been the subject of numerous theoretical and experimental investigations. The theoretical studies normally suffer due to their complexity, or have aimed at definition of only certain aspects of the separation phenomena, e.g., the pressure rise across the separation. Experimental approaches have revealed Mach number, Reynolds number, and heat transfer effects on the extent of separation.

This report presents a unified approach to the boundary layer separation problem, including a simple prediction technique for the extent of the separation, as well as the effects of initial flow parameters.

SECTION II

ANALYSIS

Approximate closed-form solutions, a simplification of the integral approach of Lees and Reeves[1], for laminar separated flows, are given in Reference 2, where the solutions contain the following terms,

$$(1 + \beta)^{1/2} = (\frac{\Delta M \theta Re_\theta}{2LK_s}) \tag{1}$$

and

$$\beta \tan^{-1} \beta = (\frac{x_{HL} - x_1}{L}) \tag{2}$$

For moderately hypersonic flows, i.e., $M_1^2 \gg 1$,

$$\Delta M = 0.2 M_1^2 \Theta_F$$

and

$$L = 0.1082 x_1 \ (-\Delta H_{tr}/\Delta K)^{1/2} \overline{x}_1^{-1/2}$$

So that Equations (1) and (2) become,

$$(1 + \beta)^{1/2} = M_1 \Theta_F / \lambda \overline{x}^{1/2} \tag{3}$$

and

$$\beta \tan^{-1} \beta = 9.242 \ (\frac{x_{HL}}{x_1} - 1) \overline{x}_1^{1/2} \ (-\Delta H_{tr}/\Delta K)^{-1/2} \tag{4}$$

where

$$\lambda = -2K_s \ [\ -(\frac{\Delta H_{tr}}{0.664 \Delta K}) \ ]^{1/2}$$

Physically β is a measure of the extent that the separation interaction is above the incipient condition, i.e., no separation, since for incipient separation, $\beta = 0$, Equation (3) yields,

$$M_1 \Theta_{F_i} = \lambda \overline{x}_1^{1/2}$$

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It is noted that the following approximately linear relation may be used,

$$\beta \tan^{-1} \beta \simeq C \ [\ (1 + \beta)^{1/2} - 1]$$

So that, with $C = 1.63$, Equations (3) and (4) may be combined to yield,

$$\frac{x_{HL}}{x_1} = 1 + 0.177 \ (-\Delta H_{tr}/\Delta K)^{1/2} \ \bar{x}_1^{1/2} \ (\frac{M_1 \Theta_F}{\lambda \bar{x}_1^{1/2}} - 1) \tag{5}$$

Figure 1 presents the factor $(-\Delta H_{tr}/\Delta K)^{1/2}$, calculated from the similar solutions of Reference 3, as well as λ, from Reference 4, as a function of T_w/T_O. The beginning of the separation interaction is then predicted by Equation (5) for given upstream conditions and flap deflection angle, Θ_F.

It is immediately recognized that Equation (5) inaccurately describes incipient separation since $x_1 \neq x_{HL}$, but this is thought to be negligible due to the definition of x_1 as the streamwise coordinate at which the tangent to the separation pressure rise at the inflection point intersects with the pressure distribution for no separation. Therefore, for incipient separation, x_1 is very close to being identical to x_{HL}.

The general validity of form and usefulness of Equation (5) must, however, be obtained by comparing its prediction for x_1 with that obtained experimentally. This comparison is given in Figure 2. The overall agreement is much better than expected, considering the simplifying assumptions involved in the analysis. The effect of including data with transition of the flow prior to reattachment is also indicated. It is seen that the theory overpredicts the extent of separation for transitional flows.

With Equation (5), it is possible to show quantitative effects of Mach number, Reynolds number, and wall temperature on the extent of separation caused by the compression surface at an angle of Θ_F. Calculations were made over a Mach number range of 4 to 8, a Reynolds number range of 10^4 to 10^6, and a wall to stagnation temperature ratio range of 0.1 to 1, for a flap deflection angle variation from the incipient angle to 25°.

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SECTION III

RESULTS AND DISCUSSION

In order to better understand the quantitative effects of the parameters of the boundary layer separation, Equation (5) is rewritten as,

$$X_1 = \frac{x_{HL} - x_1}{x_1} = K_0 M_1 \Theta_F - K_2 \frac{M_1^{3/2}}{Re_1^{1/4}} \tag{6}$$

where $K_2 = \lambda K_0 = 0.177 \, (-\Delta H_{tr}/\Delta K)^{1/2}$, i.e., K_0 and K_1 are only functions of the wall to stagnation temperature ratio and X_1 is the length of the separated region as measured from the hinge line, x_{HL}, in terms of the distance from the leading to the beginning of the interaction, x_1. Although this representation of the extent of separation is somewhat cumbersome and non-standard, Equation (6) does yield the following directly, when the relation for the incipient compression surface angle is recalled,

$$X_1 = K_0 M_1 \, (\Theta_F - \Theta_{F_i}) = K_0 M_1 \, (\Delta \Theta_F) \tag{7}$$

where Θ_F and Θ_{F_i} are in radians.

The term $\Delta \Theta_F$ in Equation (7) may be thought of, from an inviscid standpoint, as a measure of the pressure rise that the boundary layer is unable to negotiate without separating. It is also interesting that the dependence of the extent of separation on Reynolds number enters only through the incipient deflection angle, Θ_{F_i}. Also, the extent of separation, in terms of X_1, is linear in $\Delta \Theta_F$ for given M_1 and T_w/T_0 and linear in Θ_F for given M_1, Re_1, and T_w/T_0. The increase in X_1 with increasing Θ_F or $\Delta \Theta_F$ is given by the product $K_0 M_1$ independent of Re_1; so that for a given T_w/T_0 and $\Delta \Theta_F$, X_1 is doubled, for example, if the initial Mach number M_1 is doubled. Similarly for given T_w/T_0, M_1 and Θ_F, X_1 increases with increasing Reynolds number by the Reynolds number to the minus one-quarter power difference. These aspects are illustrated in Figures 3 and 4 at a wall to stagnation temperature ratio of one.

It should be noted in Figure 4 that two distinct regions exist for the effect of Mach number on the extent of separation, when the compression angle, Θ_F, is used as a base. Holding the Reynolds number and wall temperature constant permits determination of incipient compression angles, Θ_{F_i} for various Mach number. Now Θ_{F_i} increases with increasing Mach number, but the rate of increase of the extent of separation also increases with Mach number, so that for a certain range of compression angles above Θ_{F_i}, the extent of separation will decrease with increasing Mach number. At larger compression angles the extent of separation increases as the Mach number is increased. These effects are masked when the difference, $\Delta \Theta_F$, is used as a base (Figure 3).

A more common method of expressing the extent of separation is to present it as a fraction of the hinge line distance, x_{HL}, i.e.,

$$\frac{x_{HL} - x_1}{x_{HL}} \equiv X_{HL} = \frac{X_1}{1 + X_1} \tag{8}$$

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So that Equation (8) relates the extent of separation to x_{HL}, e.g., if $X_{HL} = 0.2$ the extent of separation is 20% of x_{HL}, or equally valid, but less commonly used, 25% of x_1. Because of the nonlinearity between X_{HL} and X_1, the relation between X_{HL} and Θ_F or $\Delta\Theta_F$ is no longer linear (Figure 5).

Also shown in Figure 5 is the decrease of the extent of separation by cooling the wall. The effect of wall temperature on the extent of separation is summarized in Figure 6 in terms of $\Delta\Theta_F$. The separation increases almost linearly as the wall temperature increases. The rate of increase in separation with wall temperature is seen to be essentially constant with Mach number and $\Delta\Theta_F$. Therefore, the difference in extent of separation between a highly cooled wall and an adiabatic wall is about 15% of x_{HL}, i.e., if 30% of the surface is separated at $T_w/T_o \simeq 0$, then about 45% will be separated at $T_w/T_o = 1$, or in general for a set of M_1 and $\Delta\Theta_F$,

$$\Delta(X_{HL}) \simeq 0.15\Delta\,(T_w/T_o).$$

SECTION IV

CONCLUSIONS

A simple technique has been developed for the prediction of the extent of separation due to a compression corner for a laminar, two-dimensional boundary layer. Agreement with experimental data was found to be good. Based on the equation for the extent of separation, the following observations may be made for the parameters governing the phenomena:

COMPRESSION ANGLE, Θ_F

The laminar boundary layer is able to negotiate a certain compression angle, Θ_{F_i}, without separating. Increasing the compression angle beyond Θ_{F_i} increases the extent of separation.

REYNOLDS NUMBER, Re_1

The Reynolds number upstream of the separation interaction enters into the problem only through the dependence of Θ_{F_i} on Re_1. As the Reynolds number increases, Θ_{F_i} decreases. It is therefore more practical to consider the difference $\Theta_F - \Theta_{F_i} = \Delta\Theta_F$ for the separation interaction. The extent of separation as a fraction of the unseparated extent, X_1, is linear in $\Delta\Theta_F$. The rate of increase of X_1 with $\Delta\Theta_F$ is directly proportional to the initial Mach number M_1.

MACH NUMBER, M_1

The incipient angle Θ_{F_i}, increases with increasing Mach number; however, the rate of increase of the extent of separation also increases with Mach number, so that for a certain range of Θ_F above Θ_{F_i} the extent of separation decreases with increasing Mach number. At higher compression angles, the separation increases as the Mach number increases.

HEAT TRANSFER, T_w/T_o

Cooling reduces the extent of separation. For a given $\Delta\Theta_F$, the separation extent, as a fraction of the hinge line distance, X_{HL}, decreases at a rate of about 15% of the change in the wall to stagnation temperature ratio.

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APPENDIX

TURBULENT BOUNDARY LAYER SEPARATION

It has been indicated previously that the dependence on Reynolds number of the extent of separation enters through the incipient compression angle, Θ_{F_i}. Therefore, it may be postulated that Equation (7), i.e., $X_1 = K_0 M_1 \Delta\Theta_F$ is applicable not only to laminar, but also to turbulent boundary layers. Each boundary layer having a different relation between Θ_{F_i} and Re_1 as follows:

$$\text{Laminar} \qquad -- \qquad \Theta_{F_i} = \lambda M_1^{1/2} Re_1^{-1/4} \tag{A1}$$

$$\text{Turbulent} \qquad -- \qquad \Theta_{F_i} = \lambda M_1^{1/2} Re_1^{-1/10}. \tag{A2}$$

The development of Equation (A2) is identical to that for Equation (A1) except that $Re_\theta \sim Re_x^{1/5}$ is used in the turbulent case rather than the $Re_\theta \sim Re_x^{1/2}$ as for the laminar case.

For moderately hypersonic speeds, very little data exist to compare the predictions given by Equations (7) and (A2), for turbulent boundary layers, with experimental results. The correct trend for turbulent boundary layers is predicted by Equation (7) in that the extent of separation will be less than for a laminar boundary layer, since Θ_{F_i} is much larger for the turbulent case, i.e., $\Delta\Theta_F$ is smaller.

The quantitative accuracy of the extent of turbulent separation as predicted by Equation (7) is, however, unknown. Some experimental data for the turbulent incipient angle are available. These data are summarized in Figure (7) along with transitional incipient data. No laminar experimental data are shown, although a large body exists, since the validity of Equation (A1) was shown in Reference 4. The extreme sensitivity of Θ_{F_i} for transitional separation to changes in Re_1 should be noted.
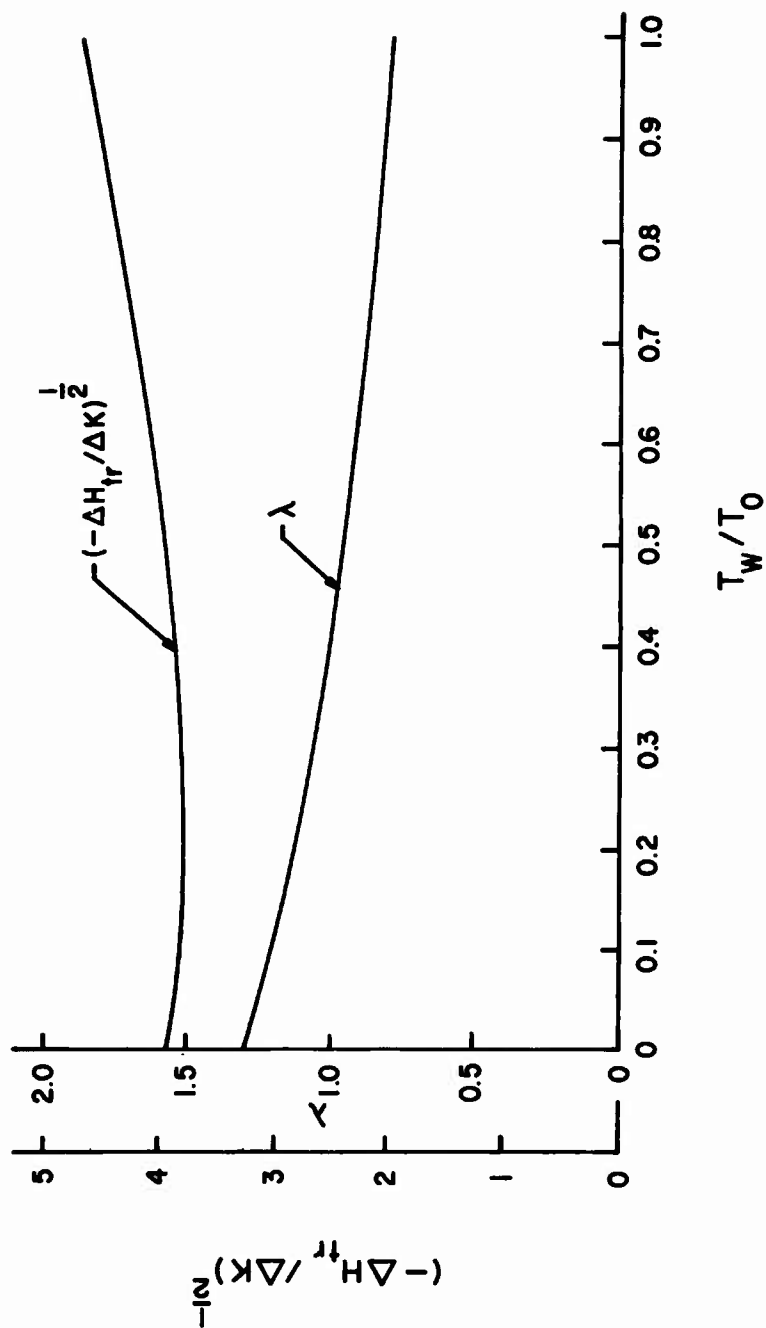
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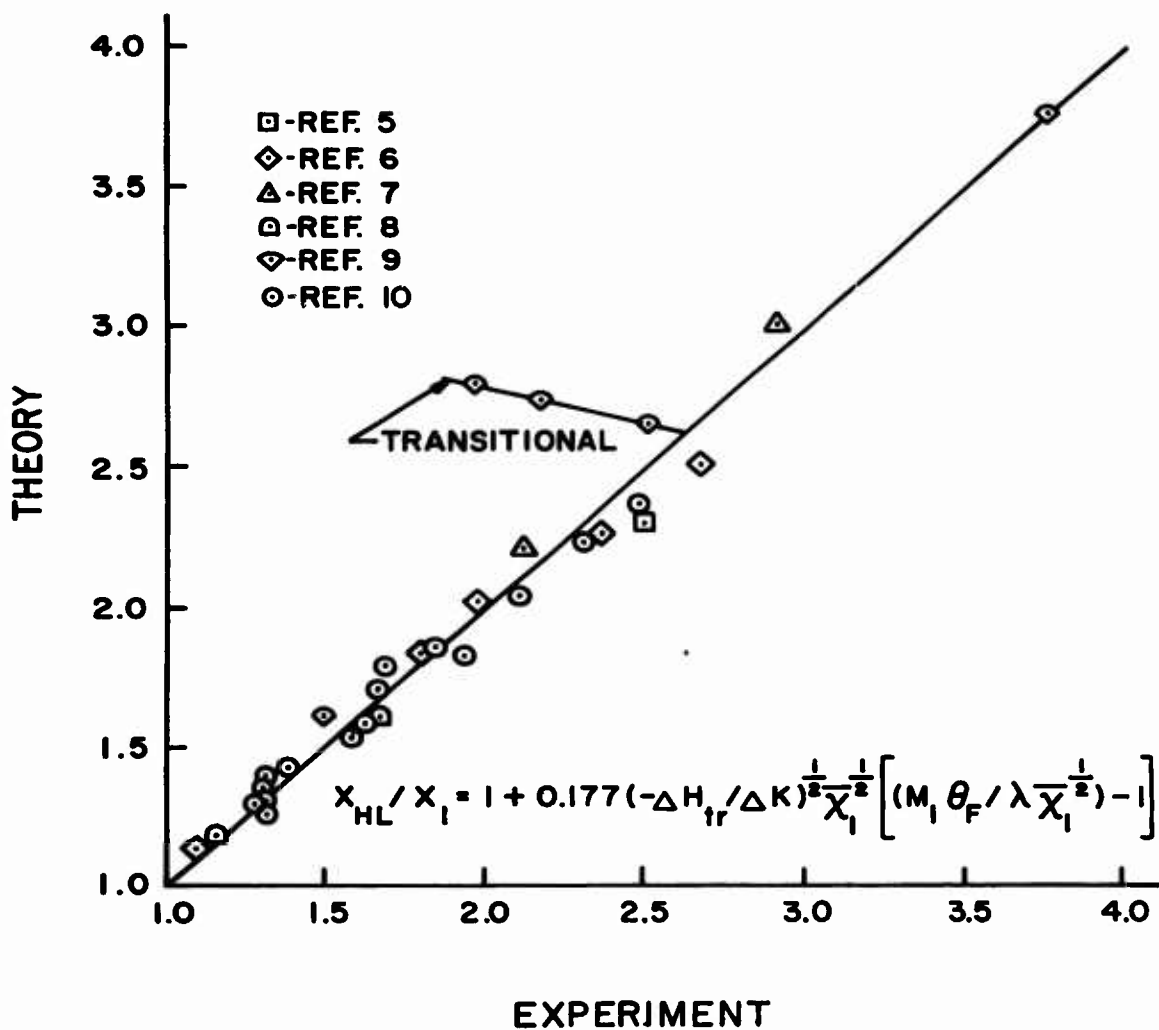Figure 1.   Variation of Similar Parameters with Wall to Wall Stagnation Temperature Ratio.

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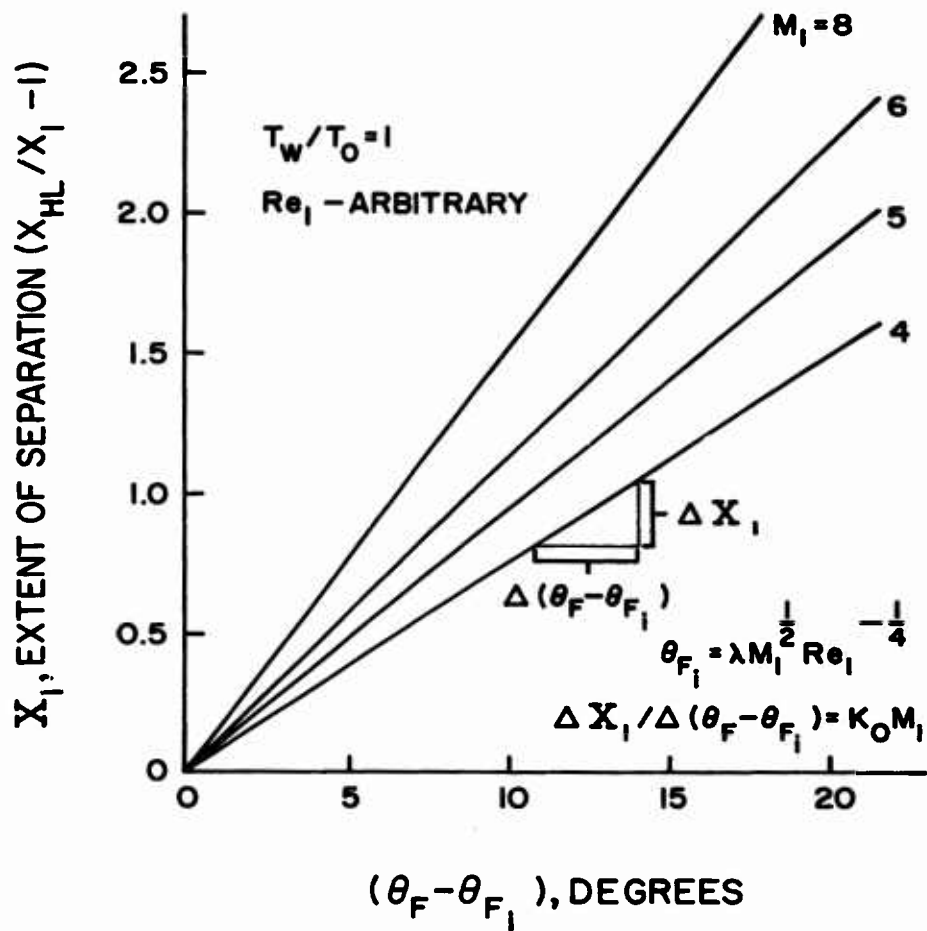Figure 2. Comparison of Theoretical Prediction of the Extent of Separation with Experiment.

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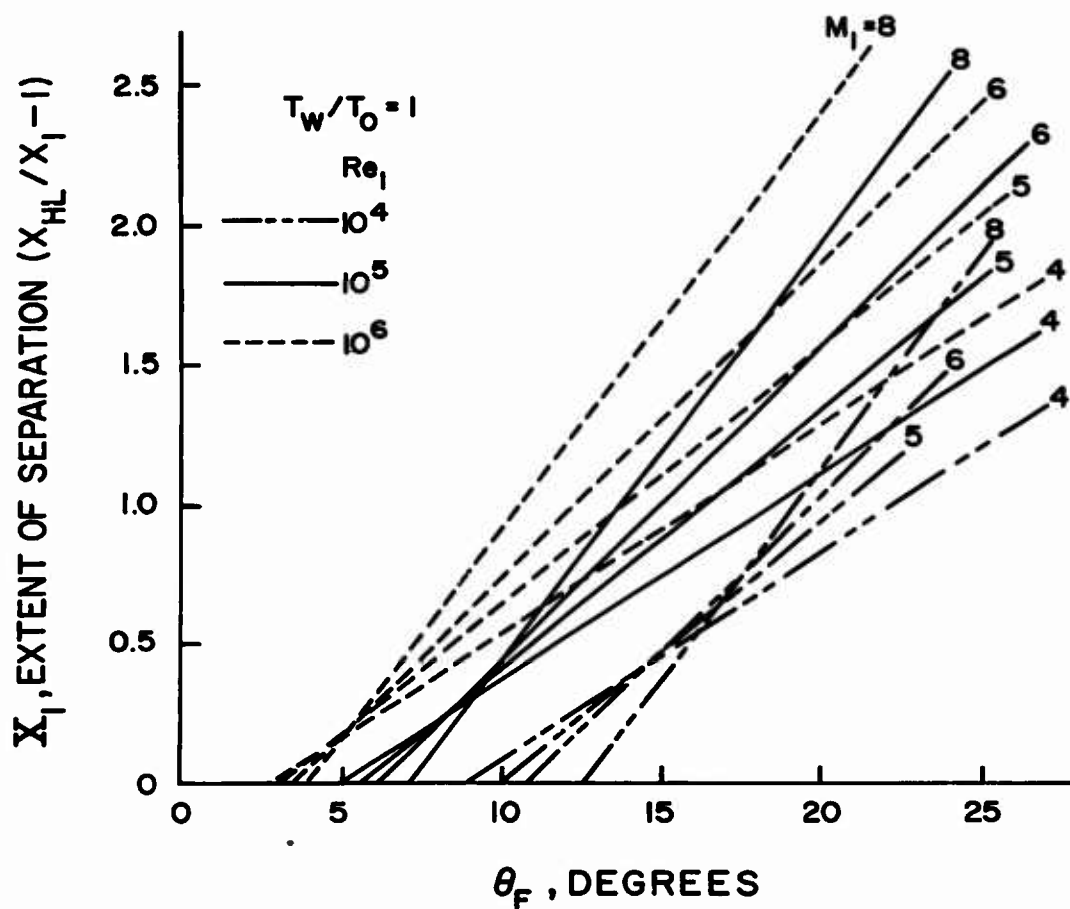Figure 3.   Mach Number Effects on the Extent of Separation for Given $(\theta_F - \theta_{F_i})$.

Figure 4. Reynolds Number Effects on the Extent of Separation for Given θ_F.
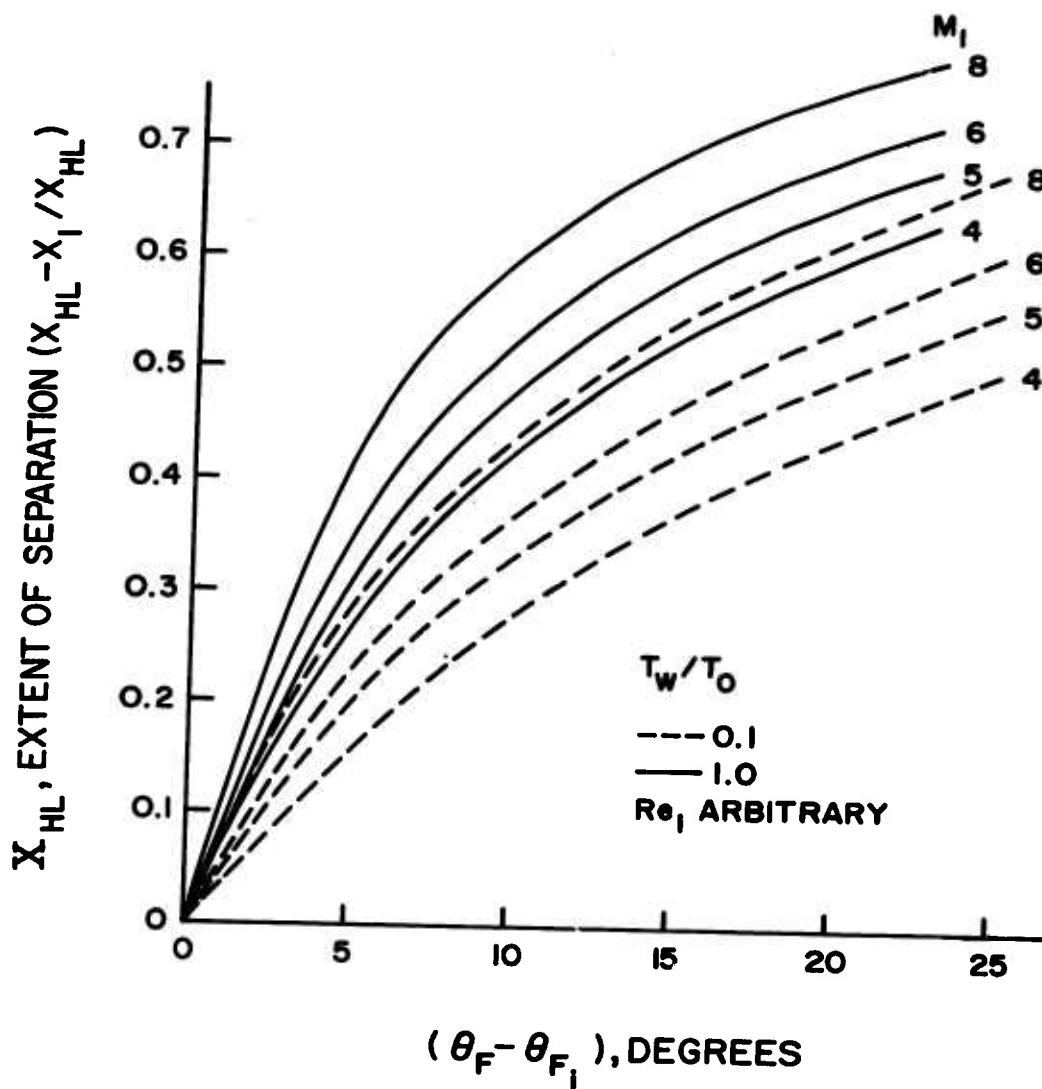
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Figure 5. Mach Number and Wall Temperature Effects on the Extent of Separation.

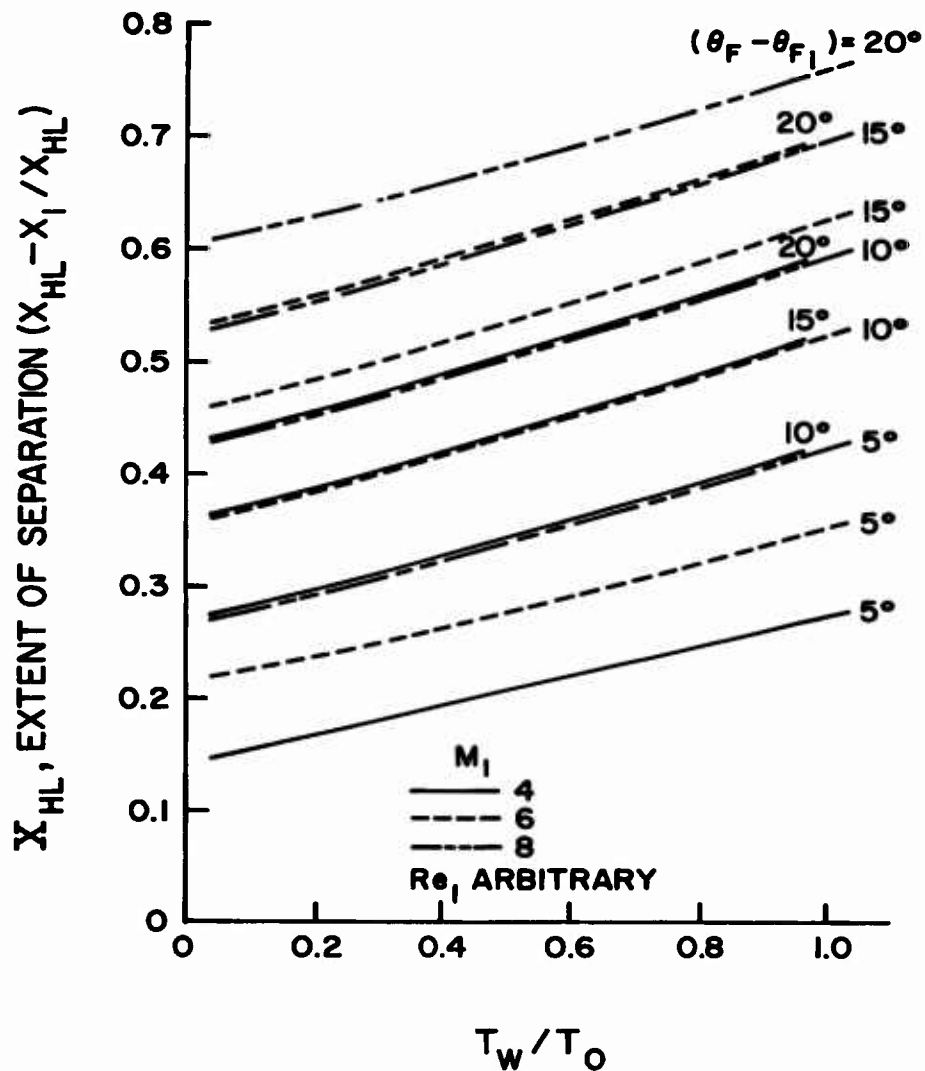Figure 6.   Summary of Wall Temperature Effects on the
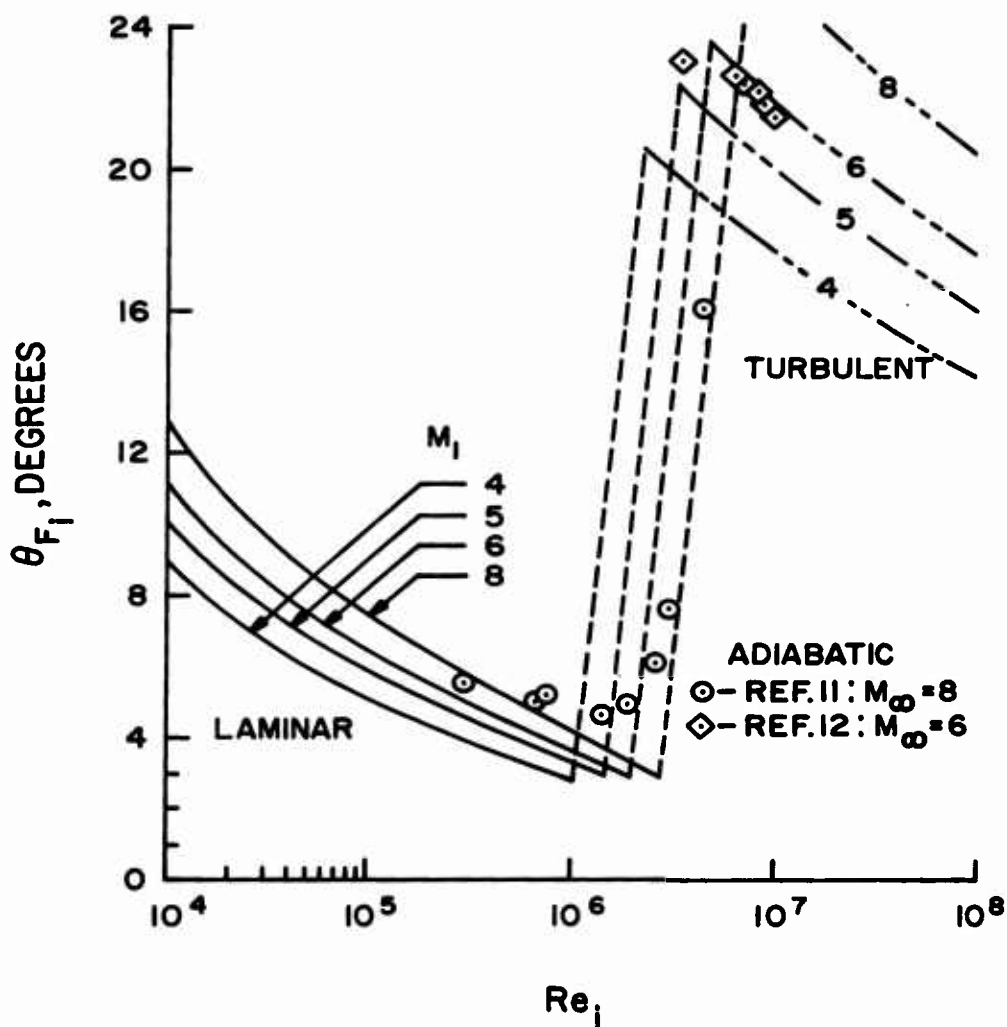   Extent of Separation.

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Figure 7.   Predicted Incipient Compression Surface
            Deflection Angles for Laminar and Turbulent
            Flows.

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